Dynamical Instability of a Doubly Quantized Vortex in a Bose-Einstein condensate

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(Dated: February 2, 2008)

Doubly quantized vortices were topologically imprinted in $|F=1\rangle$ ^{23}Na condensates, and their time evolution was observed using a tomographic imaging technique. The decay into two singly quantized vortices was characterized and attributed to dynamical instability. The time scale of the splitting process was found to be longer at higher atom density.

PACS numbers: 03.75.Kk, 03.75.Lm, 67.90.+z

Quantum fluids, like superfluid He, electrons in a superconductor or a Bose-Einstein condensate of atoms, are described by a macroscopic wavefunction. This requires the flow field to be irrotational, and gives rise to superfluidity and quantized circulation [1]. Atoms in a Bose-Einstein condensate, for example, can only circulate with angular momentum equal to integer multiple of $\hbar$, in the form of a quantized vortex [2].

Vortices are excited states of motion and therefore energetically unstable towards relaxation into the motional ground state, where the condensate is at rest. However, quantization constrains the decay: a vortex in Bose-Einstein condensates cannot simply fade away or disappear, it is only allowed to move out of the condensate or annihilate with another vortex of opposite circulation. Vortex decay and metastability, due to inhibition of decay, have been a central issue in the study of superfluidity [3, 4, 5, 6, 7, 8]. In almost pure Bose-Einstein condensates, vortices with lifetimes up to tens of seconds have been observed [9, 10, 11].

Giving a Bose-Einstein condensate angular momentum per particle larger than $\hbar$ can result in one multiply-quantized vortex with large circulation or, alternatively, in many singly-quantized vortices each with angular momentum $\hbar$. The kinetic energy of atoms circulating around the vortex is proportional to the square of the angular momentum; therefore the kinetic energy associated with the presence of a multiply-quantized vortex is larger than the kinetic energy of a collection of singly-quantized vortices carrying the same angular momentum. A multiply-quantized vortex can decay coherently by splitting into singly-quantized vortices and transferring the kinetic energy to coherent excitation modes, a phenomenon called dynamical instability which is driven by atomic interactions [5, 12, 13, 14], and not caused by dissipation in an external bath. Observations of arrays of singly-quantized vortices in rapidly rotating condensates [10, 11] indirectly suggests that the dynamical instability leads to fast decay of multiply-quantized vortices. However, the existence of stable multiply-quantized vortices in trapped Bose-Einstein condensates has been predicted with a localized pinning potential [12] or in a quartic potential [15]. Stable doubly-quantized vortices were observed in superconductors in presence of pinning forces [16] and in superfluid ^{3}He-A which has a multicomponent order parameter [17]. Recently, formation of a multiply-quantized vortex in a Bose-Einstein condensate has been demonstrated using topological phases [18, 19], and surprisingly long lifetime of a "giant" vortex core has been reported [20]. The study of topological excitation and their stability is an active frontier in the field of quantum degenerate gases [21, 22].

In this Letter, we study the time evolution of a doubly-quantized vortex state in a Bose-Einstein condensate, and directly confirm its dynamical instability by observing that a doubly-quantized vortex core splits into two singly-quantized vortex cores. The characteristic time scale of the splitting process was determined as a function of atom density and was longer at higher atomic density.

Bose-Einstein condensates containing over 10^7 ^{23}Na atoms were created in the $|F=1, m_F=-1\rangle$ state, transferred into an auxiliary chamber [23], and loaded into a Ioffe-Pritchard magnetic trap generated by a microfabricated atom chip [24, 25, 26]. The wire pattern on the atom chip is shown in Fig. 1(a). In our previous work, we used a Z-shaped wire trap where changing the sign of the axial magnetic field curvature was technically impossible so that we could not trap condensates after imprinting a vortex. To overcome this technical difficulty, we designed our new chip with separate end-cap wires, allowing independent control of the axial magnetic field. Typical wire currents were $I_C = 1.53$ A in the center wire and $I_L = I_R = 0.1$ A in the end-cap wires, and the external magnetic field was $B_z = 450$ mG and $B_x = 5.3$ G, resulting in a radial (axial) trap frequency $f_r = 220$ Hz ($f_z = 3$ Hz) and a distance of the trap from the chip surface $d = 600$ μm. After holding condensates for 2 s to damp excitations which might have been caused by the loading process, condensates contained over 1.5×10^6

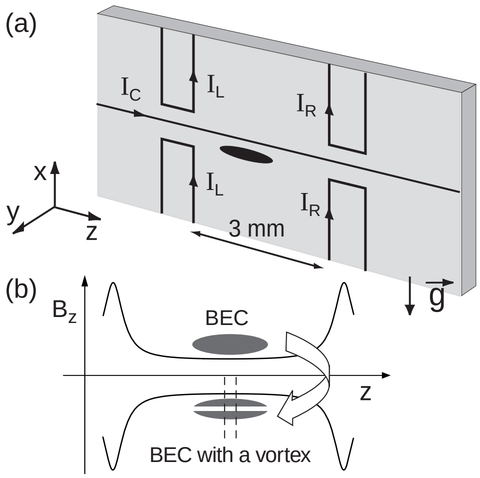

FIG. 1: (a) Wire pattern on the atom chip. A magnetic trap is formed by a current I_C flowing through the center wire in conjunction with a external uniform magnetic field B_x. The axial confinement along z direction is generated by currents I_L and I_R in the end-cap wires. Each current is controlled independently. A 2 μm thick Au film was deposited by evaporation on a thermally oxidized Si substrate and wires were patterned by photolithography and wet etching. The width of the center wire and the end-cap wires were 50 μm and 100 μm, respectively. (b) Imprinting of a vortex in a Bose-Einstein condensate. By inverting the z direction magnetic field B_z, a doubly quantized vortex was imprinted in $|F=1\rangle$ condensates, using topological phases as in Ref. [19]. The direction of I_L and I_R were also reversed to maintain the axial confinement. The dashed lines indicate the selective probing region for tomographic imaging as described in the text.

atoms and the lifetime of condensates was ≈ 8 s with a radio-frequency (rf) shield [27].

Doubly-quantized vortices were topologically imprinted in condensates by inverting the axial magnetic field, B_z, as demonstrated in Ref. [19]. B_z was ramped linearly from 450 mG to -460 mG in 12 ms. As B_z passed zero, the sign of axial field curvature was changed by reversing the directions of I_L and I_R in 1 ms. The trap center position and the axial trap frequency of the inverted wire trap were matched to those of the original wire trap by adjusting the final values for I_L and I_R. Losses due to nonadiabatic spin flips as B_z passed through zero reduced the number of atoms in the condensate after imprinting to about $\sim 1\times10^6$, giving a typical healing length $\xi = 0.4$ μm. The lifetime of condensates after imprinting was less than 2 s.

The vortex imprinting process was accompanied by a sudden mechanical squeeze in the radial direction and a kick in the vertical direction. The radial trap frequency is proportional to the square root of the bias magnetic field ($f_r \propto |B_z|^{-1/2}$) and became temporarily higher during field inversion. Additionally, the vertical position of the trap center changed as the gravitational sag ($\propto f_r^{-2}$)

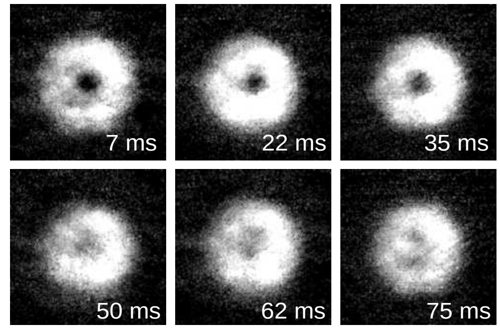

FIG. 2: Decay of a doubly quantized vortex. Axial absorption images of condensates after 15 ms of ballistic expansion with a variable hold time after imprinting a doubly quantized vortex. A doubly quantized vortex decayed into two singly quantized vortices. For this data, the interaction strength was $an_z \approx 7.5$ (see text for definition). The field of view in each image is 320 μm $\times$ 320 μm.

changed from 5.1 μm to zero. The Thomas-Fermi radius of condensates in the loading phase was ~ 5 μm. After imprinting a vortex, the amplitude of quadruple oscillation in the axial direction was ~ 20 % of the axial length of condensates (≈ 600 μm), but there was no detectable dipole oscillation in the vertical direction.

The decay of a doubly-quantized vortex state was studied by taking an absorption images along the imprinted vortex line after releasing the condensate and letting it expand for 15 ms. When we took an integrated absorption image, the visibility of a vortex core completely vanished within 30 ms. To reduce blurring due to possible bending of the vortex line [28], we employed a tomographic imaging technique [29]. A 30 μm thick central slice of the condensate (see Fig. 1(b)) was selectively pumped into the $F=2$ hyperfine level with a sheet of laser light perpendicular to the condensate long axis; the radial profile of the condensate in the selected region was then imaged with a light pulse resonant with the $F=2 \rightarrow F'=3$ cycling transition. In our absorption images, the size of a doubly-quantized vortex core was typically ~ 40 μm. This tomographic imaging technique was crucial for observing the time evolution of vortex cores beyond 30 ms.

A series of absorption images of the splitting process is provided in Fig. 2. Images taken just after imprinting show a doubly-quantized vortex core of high visibility; the visibility of the core decreased with time, an effect we attribute to bending of the vortex line [28] and other excitations created during the imprinting process. Later in the evolution, the central core deformed into an elliptical shape and split into two closely-spaced cores. Once the two cores were separated by their diameter, they appeared well resolved in our images. The angular position of the two cores was random for each experimental realization with the same evolution time, so the precession

frequency of two cores could not be determined with our destructive image technique.

To investigate the dependence of the instability on the mean field atomic interaction, we measured the characteristic time scale of splitting of a doubly-quantized vortex core as a function of the atom density. Atom density was controlled by removing a variable number of atoms with rf evaporation before imprinting a vortex. Images were classified as follows: images where the two cores were separated by more than one core diameter were labelled as "two visible cores"; images with a clearly-defined circular central core were labelled as "one core"; images in the intermediate range, where the central core was elliptical but the two cores were not resolved, or with a bad visibility were labelled as "undetermined". For example, the images at 62 ms and 75 ms in Fig. 2 and Fig. 3(a) were classified as "two visible cores", and 50 ms in Fig. 2, and Fig. 4(a) and (c) as "undetermined".

Experimental results are provided in Fig. 3 as a function of the linear atom density n_z (along the condensate long axis) multiplied by the s-wave scattering length a. The rescaled quantity, $an_z = a \int |\psi(r)|^2 dxdy$ corresponds for a cylindrical condensate to the strength of the mean field interaction, with $\psi(r)$ being the condensate wavefunction. Results in Fig. 3 clearly demonstrate that a doubly-quantized vortex core splits more slowly as the density becomes higher.

Once the doubly-quantized vortex core split into two cores, the distance between the two cores was almost constant (~ 50 μm) during the further evolution, as shown in Fig. 3(c). This is evidence that the separation process was driven mainly by the dynamical instability, and not by dissipation, which would gradually increase the separation of the two cores. Dissipative processes were minimized by performing the experiments at the lowest possible temperature. Condensates did not have any discernible thermal atoms even after extended hold time. Furthermore, the energy released by the dissociation of the doubly-quantized vortex was ~ 5 nK negligible to the critical temperature ~ 240 nK. For the upper bound to the temperature of < 100 nK, Ref. [30] predicts that dissipative decay time to be ≈ 1.5 s for a single vortex, a time scale much longer than what we observed.

Multiply-quantized vortices in a harmonic potential are predicted to spontaneously decay into other states even in the absence of dissipation and external perturbations [5]. In the Bogoliubov framework, which is believed to well describe quantized vortices in one component condensates, the dynamical instability manifests as the existence of excitation modes with a complex eigenfrequency. The nonvanishing imaginary part of the eigenfrequency implies an exponential growth in time of the corresponding excitation mode, leading to decay of the multiply-quantized vortex state. This spectral instability is a general parametric phenomenon occurring when several modes compete during coherent evolution and has

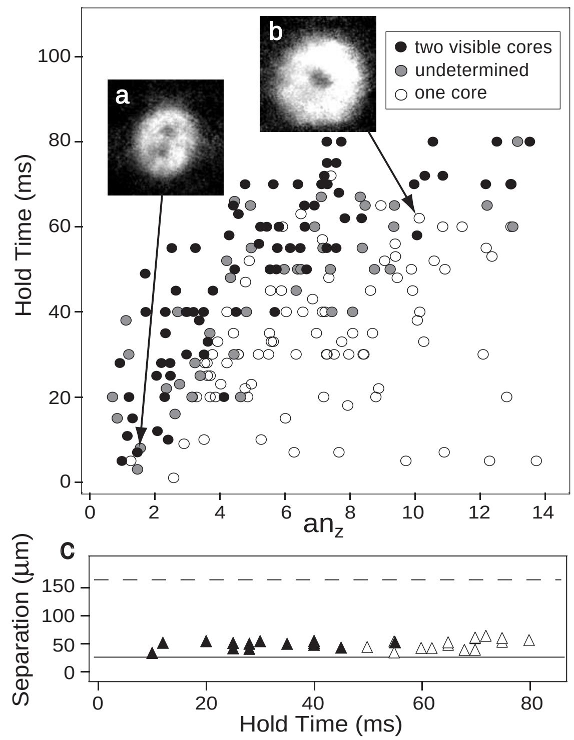

FIG. 3: Density dependence of the decay process. The time scale for the decay process of doubly quantized vortex states was measured by observing the vortex cores and classifying them as one vortex (open circles) or two vortices (solid circles). Data were collected with three axial trap frequencies $f_z = 2.7$, 3.7, 12.1 Hz and the interaction strength an_z was controlled by changing the atom number by rf induced evaporation before imprinting. Typical absorption images for (a) fast decay at low density ($an_z = 1.5$) and (b) slow decay at high density ($an_z = 10.1$). The field of view in the absorption images is 300 μm $\times$ 300 μm. (c) The separation of two visible cores vs. the hold time for $2 < an_z < 3$ (solid triangles) and $6 < an_z < 8$ (open triangles). The solid and dashed lines indicate the diameter of one vortex core and of the condensate, respectively.

been studied in many other nonlinear physical systems (see, *e.g.*, Ref. [31, 32] and references therein).

For a doubly-quantized vortex state in a cylindrically symmetric condensate, it was theoretically found that there are two excitation modes with a complex eigenfrequency [5, 13]. One of them is confined inside the doubly-quantized vortex core; the growth of this so-called "core" mode induces splitting of the original doubly-quantized vortex core into two separate singly-quantized vortex cores. The other mode, having the conjugate eigenfrequency, grows with the core mode in order to conserve energy. In the low density limit, this mode corresponds to the co-rotating quadrupole mode, leading to oscillations in the surface shape of condensates. We always observed that the surface of condensates changed into a quadrupole shape as the two cores appeared, as shown in

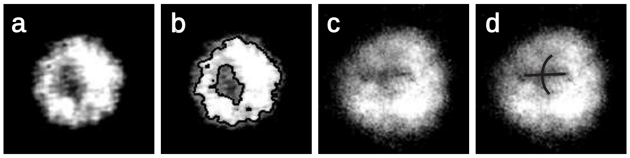

FIG. 4: Examples for the dynamic evolution after imprinting a doubly quantized vortex: (a) Surface Exicitation. Regular density modulation of the surface was observed after 51 ms hold time for $an_z = 1.8$ (b) same as (a) with a contour line. (c) Crossing of vortex lines. 55 ms hold time and $an_z = 8.4$. (d) same as (c) with guide lines for vortex lines. The field of view is 270 μm × 270 μm.

Fig. 3(a), and the ellipticity was larger at lower density.

The dynamical instability of the doubly-quantized vortex state is related to the magnitude of the imaginary part of the complex eigenfrequency, and, according to the numeric calculation in Ref. [13], nonvanishing imaginary part of the eigenfrequency appears at $an_z < 3$ and $an_z \sim 12$, showing a quasi-periodic behavior as a function of the interaction strength, an_z. The experiment showed a monotonic increase of the lifetime with no hint of periodic behavior. However, the calculated instability is not directly comparable to the observed lifetime. The imaginary part represents only the initial instability. Our criterion for decay was the observation of two *separated* vortex cores. It is possible that the dynamical instability changes after the doubly-quantized vortex state is significantly perturbed [7, 14]. It would be helpful to have more inclusive calculations leading to a lifetime directly comparable with the experiments.

What is the further evolution of the two cores? Some of the images at low density showed a regular surface modulation, as in Fig. 4(a), which was not seen in clouds with a single core. This indicates that higher-order surface modes are excited during the coherent evolution [33]. However, their reproducibility was insufficient for a systematic study. Several images, especially those labelled as "undetermined", suggest that vortex lines crossed [13, 34], as in Fig. 4(c). In our system, it was difficult to trace the positions of the two cores beyond 80 ms hold time.

In conclusion, we observed how a doubly-quantized vortex splits into a pair of singly-quantized vortices, and found higher stability at higher atom density. The topological phase imprinting technique is unique in generating doubly- or quadruply-quantized vortex states [19, 35]; a key feature is the rapid preparation of well-determined vortex states which gives access to their dynamical instabilities and coherent evolution.

This work was funded by ARO, NSF, ONR, and NASA. M.S. acknowledges additional support from the Swiss National Science Foundation and C.S. from the Studienstiftung des deutschen Volkes. We thank M. Möttönen and K. Machida for helpful discussions.

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